Bootstrapping One-Loop Amplitudes

Or: Needles and Large HaystaCks

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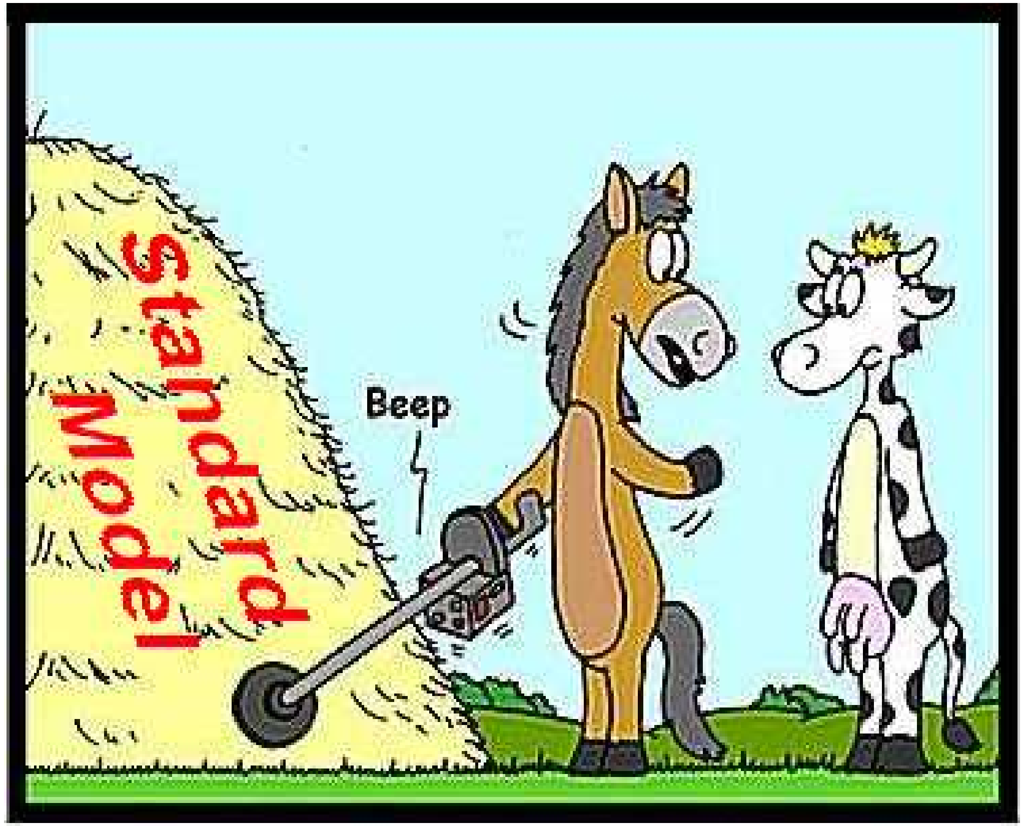
BNL 2007: New Horizons at Colliders – May 31st, 2007

Outline

- Introduction
- On-shell bootstrap at one loop – cut parts (= (poly)logarithmic terms) from generalized unitarity
- On-shell bootstrap at one loop – rational terms
- Summary and outlook
  The wishlist will get done

What the LEP, Tevatron, . . . told us



You were right: There's a needle in this haystack...

SUSY search - missing E_T+ jets

Introduction

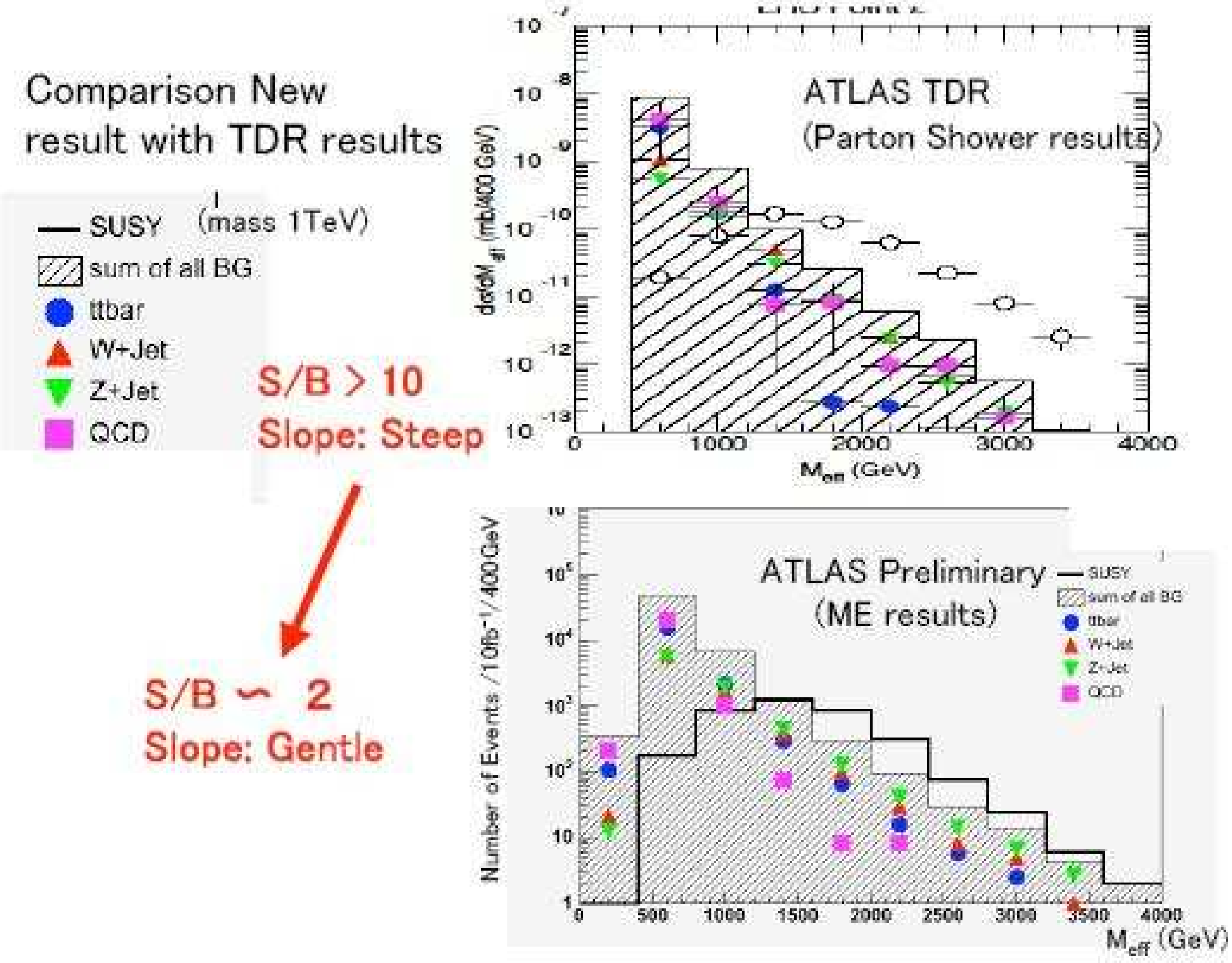
- Particle Physics in the 20th Century
- SUSY search
- Particle Physics in the 21st Century?
- The LHC Wishlists
- On-Shell Recursion Relations
- Proof at Tree-Level
- QCD at One Loop - A Disaster?

The Bootstrap Method – Generalized Unitarity

The Bootstrap Method – Rational Terms

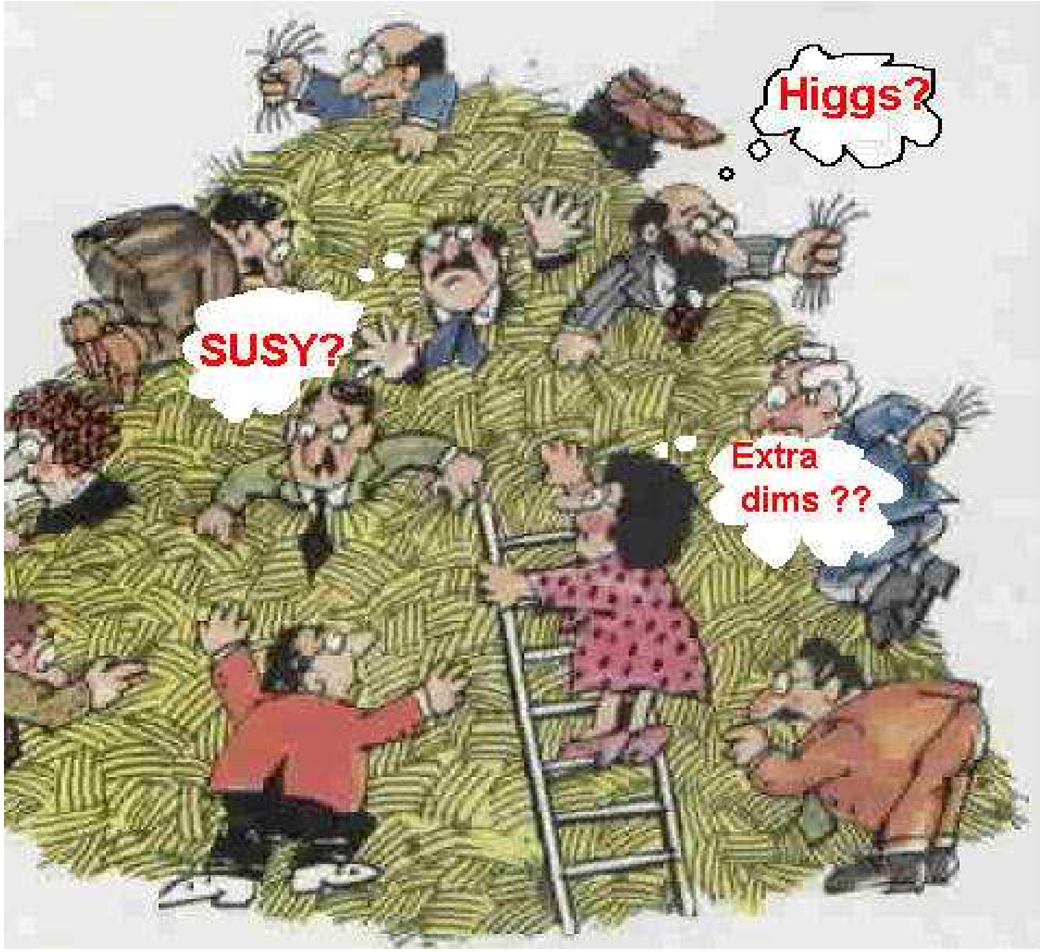
Summary and Outlook

Asai, ATLAS Rome meeting 2006



Need to understand the “haystack” to find the needle!

Physics at the LHC . . .?



The (In)Famous Experimenters' Wishlists

Run II Monte Carlo Workshop 2001

Single boson	Diboson	Triboson	Heavy flavor
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}+\leq 3j$	$WW+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}+\leq 3j$	$WW+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$ZZ+b\bar{b}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{b}+\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$		$tb\bar{b}+\leq 3j$
$\gamma+b\bar{b}+\leq 3j$	$\gamma\gamma+b\bar{b}+\leq 3j$		
$\gamma+c\bar{c}+\leq 3j$	$\gamma\gamma+c\bar{c}+\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}+\leq 3j$		
	$WZ+c\bar{c}+\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

The (In)Famous Experimenters' Wishlists

Les Houches 2005

process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

The (In)Famous Experimenters' Wishlists

Les Houches 2005

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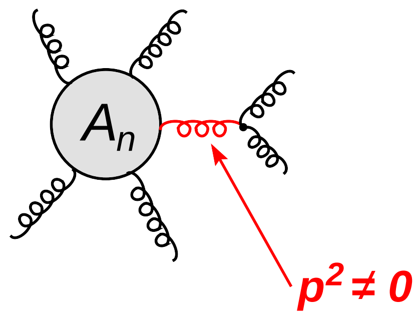
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

Large number of high-multiplicity processes that need to be computed!
The LHC turns on soon!

On-Shell Recursion Relations at Tree Level

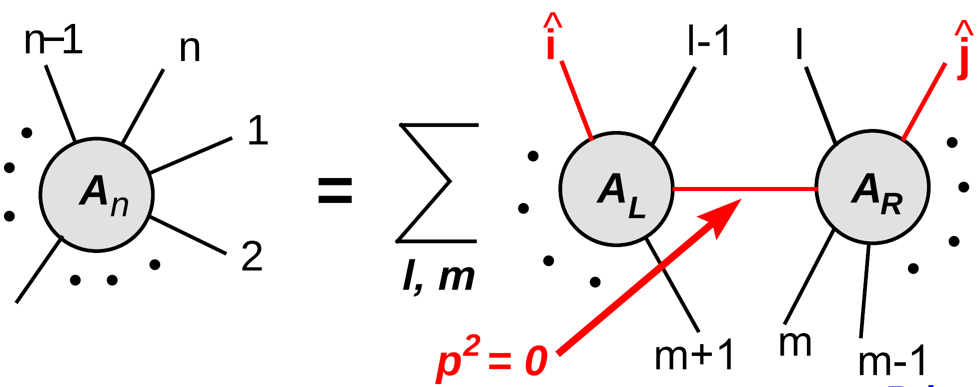


Complex continue (shift) spinors and momenta:

$$p_i \longrightarrow p_i(z) \qquad p_j \longrightarrow p_j(z)$$

$$p_i + p_j \longrightarrow p_i + p_j$$

Momentum conservation is maintained, momenta on-shell ($p_i(z)^2 = p_j(z)^2 = 0$).



Britto, Cachazo, Feng

Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

$$1/P^2_{l\ldots j\ldots m} \quad \rightarrow \quad 1/P^2_{l\ldots j\ldots m}(z)$$

$$A(z) = \sum_{l,m}\sum_{h} A^h_L(z)\frac{1}{P^2_{l\ldots j\ldots m}(z)}A^{-h}_R(z)$$

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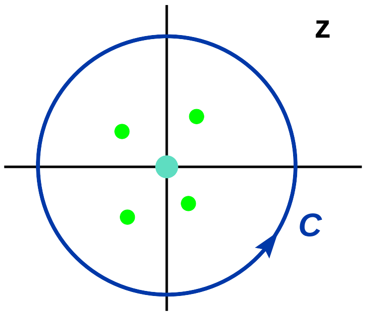
$$A(z) = \sum_{l,m}\sum_{h} A_L^h(z)\frac{1}{P^2_{l\ldots j\ldots m}(z)}A_R^{-h}(z)$$

If $A(z\rightarrow\infty)\rightarrow 0$ **- Cauchy's theorem**



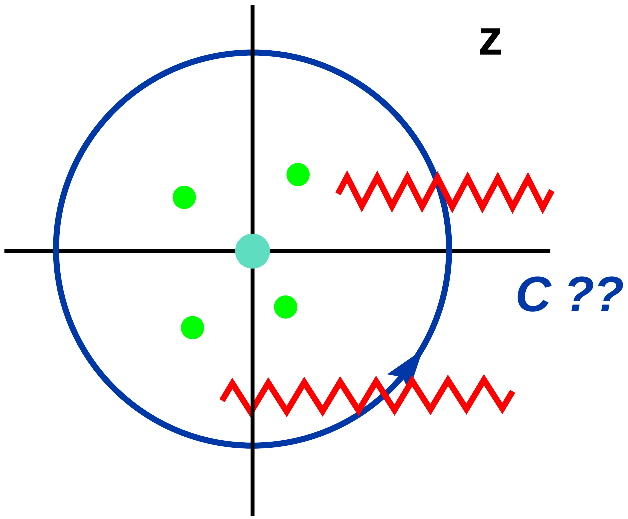
$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}A(z) = 0$$

$$A(0) = -\sum_{\text{poles }\alpha}\operatorname*{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

$$= \sum_{\text{poles }\alpha}\sum_{h} A_L^h(z_\alpha)\frac{1}{P^2_{l\ldots j\ldots m}}A_R^{-h}(z_\alpha)$$

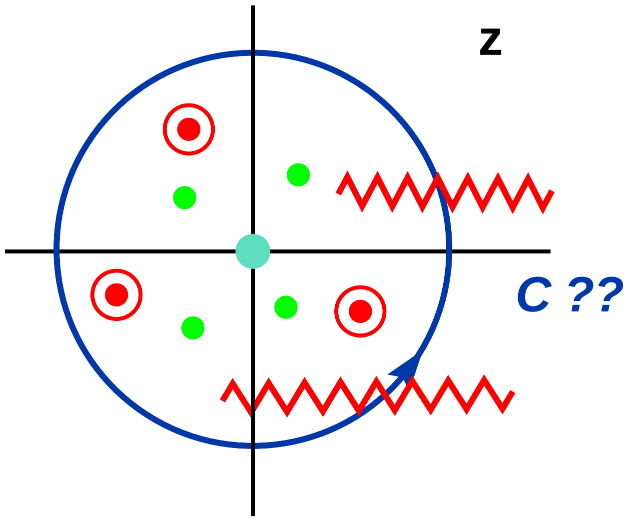
Britto, Cachazo, Feng, Witten

QCD at One Loop - A Disaster?



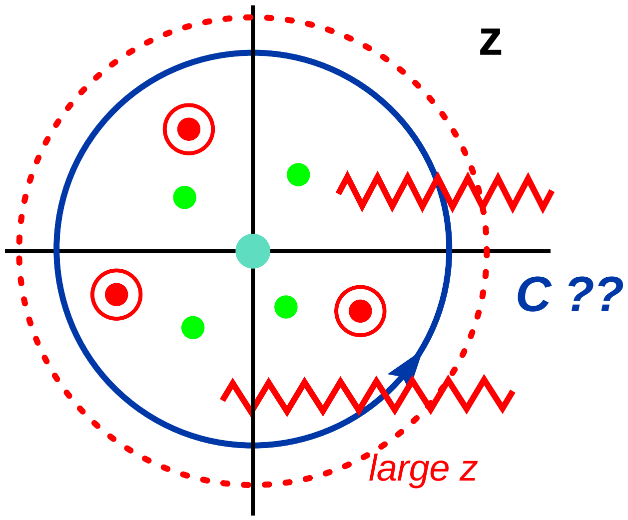
- Branch cuts (with spurious singularities)

QCD at One Loop - A Disaster?



- **Branch cuts (with spurious singularities)**
- **Double poles, 'unreal poles' and nonstandard factorizations**

QCD at One Loop - A Disaster?



- Branch cuts (with spurious singularities)
- Double poles, 'unreal poles' and nonstandard factorizations
- $A(z \to \infty) \neq 0$

On-Shell Bootstrap Method

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Bootstrap model

From Wikipedia, the free encyclopedia

In physics, the term **bootstrap model** is used for the class of theories that assume that very general consistency criteria are sufficient to determine the whole theory completely. In such theories, typically examples of quantum field theory, it is impossible to divide the objects and concepts to elementary and composite ones. See Geoffrey Chew. This strategy turned out to be successful only in the case of two-dimensional conformal field theory where many insights can indeed be derived by this method.

Here: very general consistency criteria

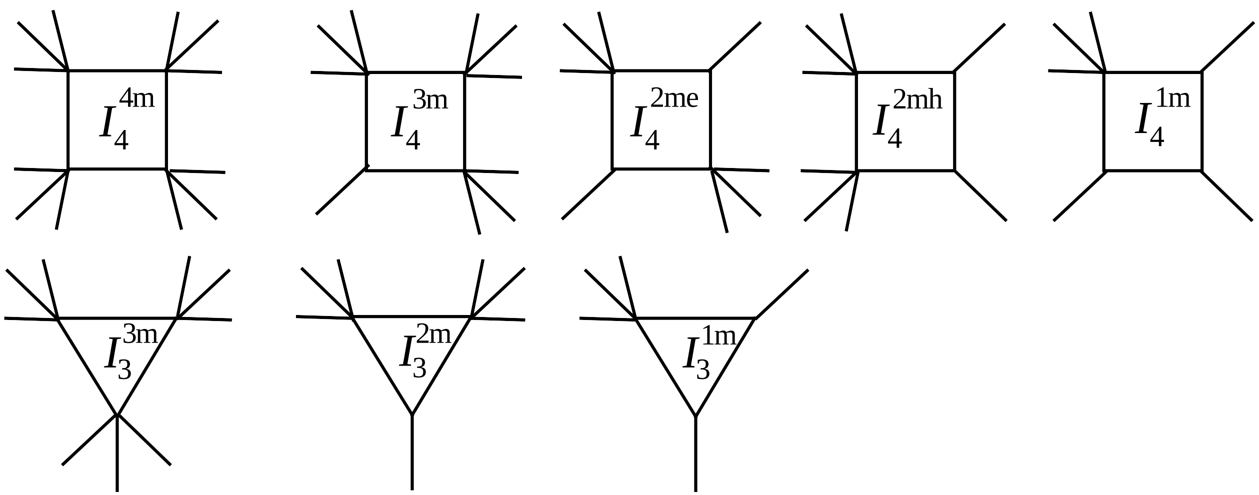
- **Cuts (unitarity)**
- **Poles (factorization)**

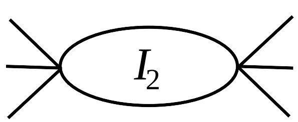
$$A(z) = C(z) + R(z)$$

Factorize independently. But: C and R talk to each other via behavior at $z \to \infty$ and spurious singularities! Need to keep this in mind when constructing recursion relations for R.

Cut Parts

$$A = \sum_{i=1}^{4} c_i I_i + R$$





rational

$$A_{n;1}^{[0]}\Big|_{1/\epsilon} = \frac{c_\Gamma}{3\,\epsilon} A_n^{\text{tree}}$$

Bern, Dunbar, Dixon, Kosower

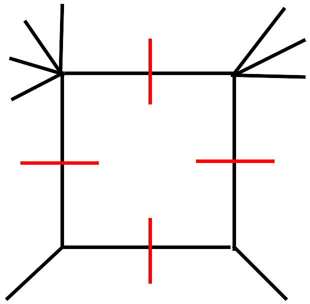
Cut Parts – Boxes

$$\frac{1}{P^2 + i\varepsilon} = \frac{1}{P^2} + i\delta^+(P^2)$$

Leading singularity of I_4 from quadruple cuts:

$$\begin{aligned}\Delta_{\rm LS} I_4 &= \int d^4 l \delta^+(l^2)\delta^+\left((l-K_1)^2\right) \\ &\times \delta^+\left((l-K_2)^2\right)\delta^+\left((l+K_4)^2\right)\end{aligned}$$

Integral completely localized, can simply read off coefficient of box integral:



$$c_4 = \frac{1}{|S|}\sum_{S,J} n_J A_1^{\rm tree} A_2^{\rm tree} A_3^{\rm tree} A_4^{\rm tree}$$

Tree graphs on shell

Trees "recycled" into loops

Britto, Cachazo, Feng

Triangles and Bubbles

Triangle coefficients from triple cuts, bubble coefficients from double cuts.
But life's not so simple – "leakage" from triangles into bubbles because integrals are not fully localized any more.
However, the singularity structures are unique – need procedure to disentangle coefficients:

- holomorphic anomaly reduces determination of coefficients to algebraic (nonlinear) manipulations without performing any integrals

  Britto, Buchbinder, Cachazo, Feng, Mastrolia

- momentum parametrization picks out relevant structure

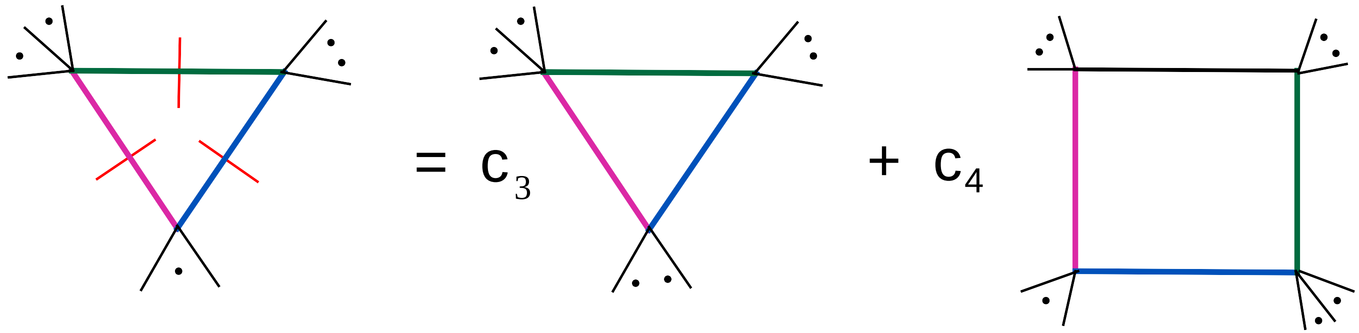
  Ossola, Papadopoulos, Pittau

Merge generalized unitarity with suitable momentum parametrization

Forde

Triangles and Bubbles contd.

Main idea



$$\int d^4 l\, \delta^+(l^2)\delta^+\left((l-K_1)^2\right)\delta^+\left((l-K_2)^2\right) A_1^{\mathrm{tree}} A_2^{\mathrm{tree}} A_3^{\mathrm{tree}}$$

$$\to \int dt\, F(t)$$

Parameterize such that (schematically)

$$\to {\color{red} c_3} \int dt + c_4 \int dt \frac{1}{t^n}$$

Bubbles have 2 free parameters, mapping not as trivial as above but still simple

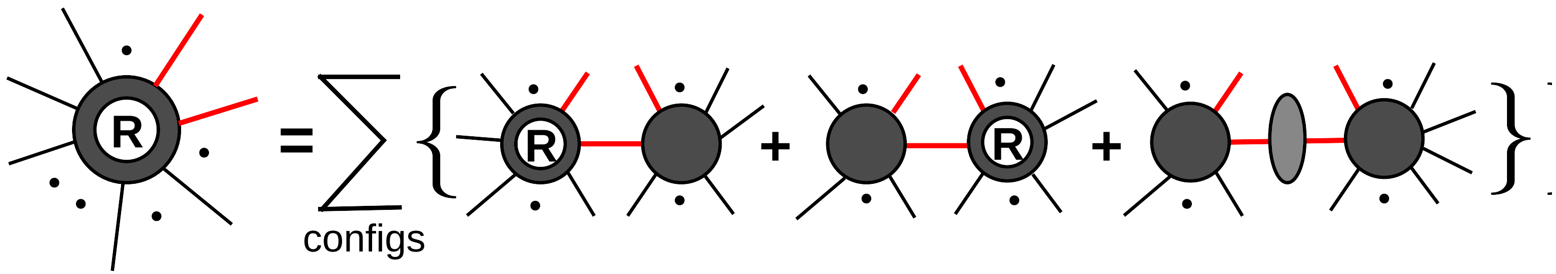
Forde

On-Shell Recursion for Rational Parts

$$A(z) = C(z) + R(z) \qquad \Bigg| \qquad \frac{1}{2\pi i}\oint_C \frac{dz}{z}$$

$$A(0) = C(0) + \mathrm{Inf}\, A - \sum_{\text{poles } \alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

$$= C(0) + \mathrm{Inf}\, A + \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$



Loops “recycled” into loops

(ignoring slight subtleties with spurious singularities)

Bern, Dixon, Kosower

Large-z Contributions

Can pick shifts to avoid either non-standard factorizations or $z \to \infty$ contributions, but in general not both!

- **Shift $[j, l\rangle$ avoids non-standard factorizations**

$$A(0) = C(0) + \operatorname*{Inf}_{[j,l\rangle} A + R^{[j,l\rangle}_{\text{recurs}}$$

Large-z Contributions

Can pick shifts to avoid either non-standard factorizations or $z \to \infty$ contributions, but in general not both!

- **Shift $[j,l\rangle$ avoids non-standard factorizations**

$$A(0) = C(0) + \operatorname*{Inf}_{[j,l\rangle} A + R^{[j,l\rangle}_{\text{recurs}}$$

- **Shift $[a,b\rangle$ has no large-parameter contributions**

$$A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard channels}^{[a,b\rangle}$$

The Bootstrap Formalism

Solution $\Rightarrow$ use two shifts!

Extract large-parameter contributions of primary shift from auxiliary relation:

$$A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard}^{[a,b\rangle} \quad \Bigg| \; [j,l\rangle \; \Bigg| \; \operatorname*{Inf}_{[j,l\rangle}$$

$$\operatorname*{Inf}_{[j,l\rangle} A = \operatorname*{Inf}_{[j,l\rangle} C + \operatorname*{Inf}_{[j,l\rangle} R^{[a,b\rangle}_{\text{recurs}}$$

if $\quad \operatorname*{Inf}_{[j,l\rangle} \left[\text{non-standard channels}^{[a,b\rangle}\right] = 0$

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$$A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard}^{[a,b\rangle} \quad \Big| \; [j,l\rangle \; \Big| \quad \operatorname*{Inf}_{[j,l\rangle}$$

$$\operatorname*{Inf}_{[j,l\rangle} A \;=\; \operatorname*{Inf}_{[j,l\rangle} C + \operatorname*{Inf}_{[j,l\rangle} R^{[a,b\rangle}_{\text{recurs}}$$

if $\operatorname*{Inf}_{[j,l\rangle} \left[\text{non-standard channels}^{[a,b\rangle}\right] = 0$

The complete bootstrap

$$\boxed{A(0) = C(0) + R^{[j,l\rangle}_{\text{recurs}} + \operatorname*{Inf}_{[j,l\rangle}\left[C + R^{[a,b\rangle}_{\text{recurs}}\right]}$$

Passes all nontrivial checks!

CFB, Bern, Dixon, Forde, Kosower

Summary

Introduction

The Bootstrap Method – Generalized Unitarity

The Bootstrap Method – Rational Terms

Summary and Outlook

- Summary
- Physics at the LHC

- ☑ Simple and automatizable method for computing cut (logarithmic) parts from tree amplitudes based on generalized unitarity — Britto, Cachazo, Feng, Forde
- ☑ Working algorithm for rational terms of all helicity configurations of one-loop gluon amplitudes — CFB, Bern, Dixon, Forde, Kosower
- ☑ All-multiplicity formulae for several one-loop gluon amplitudes (also with a fermion pair and Higgs) — CFB, Bern, Dixon, Del Duca, Forde, Kosower
- ❑ Practical issues – fermions, masses, best way to deal with spurious singularities
- ❑ Automatization – procedure seems numerically very stable
- ❑ Attack the wishlists...

Physics at the LHC